SINGLE-LAYER SPHERICAL GEODESIC DOMES INTERACTING WITH A NETWORK OF INFLATABLE BEAMS

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Summary. In this paper we propose an example of a thin geodesic spherical dome composed by a lattice of inflatable interacting beams. The geometry of the lattice is determined by searching for the minimum variation of the characteristic dimensions of the elements that make up the dome (length of the bars and area of the panels). A first illustration of the mechanical response of the structure is given, with reference to the loads that usually are assumed to act on this type of shell.

1 INTRODUCTION

In single layer spherical lattice shells, the simpler and more widespread arrangement of the structural elements consists in placing the bars along the sides of a regular triangular mesh drawn on the surface. The reasons in favour of this type of network are to be found primarily in its high structural performance; in fact, these shells allow for structures characterized by high stiffness, by conveniently exploiting the very low in-plane compliance possessed by each single triangular mesh.

Other factors that are usually considered to be of secondary importance, can gain a certain importance in the choice of the type of lattice. Among them, the possibility of using prefabrication techniques for the realization of the various structural elements is certainly the most important (Makowski, [1]). The shells lattices, as happens for all spatial structures, are constituted, in fact, from a large number of different parts (joints, bars, cladding panels). To this regard, to be able to standardize as much as possible the size of each component allows for a significant reduction of the costs of implementation (Tarnai, [2]).

In this perspective, the use of cylindrical inflatable beams instead of the usual metallic elements provides numerous benefits ([3], [4]). First of all, it is possible to achieve a reduction, which can be considerable in some cases, of the total weight of the structure. Secondly, the pre-tension that is established in the beams during their inflation phase assures that each panel of the shell is in a state of traction, thereby reducing the risk that the beams may undergo to phenomena of loss of stability of the equilibrium configuration. For these reasons, the shells obtained from the assembly of spherical lattice inflatable beams seem to have a good chance of being profitably used for building structures, both permanent (for these applications, the use of materials called "*rigidifiable*" looks promising) and temporary.

When using inflatable beams, one has to reconsider the whole organization of the design and realization of these structures. In particular, the presence of inflatable elements, which acquire stiffness by virtue of their internal pressure, and the assembly of the structure, for which no external equipment is needed (self-deployment structures), represent as many peculiar characteristics to be properly taken into account from the beginning of the design process (Comte et al. [5]).

In this work we describe an example of a spherical geodesic dome to be implemented through a network of interacting inflatable beams. The geometry of the lattice is determined by trying to minimize the variation of the characteristic dimensions of the elements that make up the dome (essentially, the length of the beams and the area of the panels). The mechanical response of the structure is discussed briefly.

2 SPHERICAL GEODESIC DOMES

The use of domes as roof structures is well documented since the ancient times. Beautiful examples of domes can be found in the buildings erected by early civilizations. Without aiming to provide a compendium of the history of the domes in civil construction, we only observe that the choice of the materials and of the particular type of structure has constantly been evolving. In a relatively short time, the typology passed from the initial thin spherical masonry domes, sometimes spherical and with slightly variable or constant thickness, requiring remarkable centrings, up to the latest and thin reticular metallic single-layer spherical domes, which do not require any kind of centring. The transition between the lattice domes with hierarchical elements and the current homogeneous ones, without any hierarchy between elements (such as, for example, the lamellar domes), has been even more rapid.

Parallel to the typological evolution, the materials used have also been changing. Driven by the need to reduce as much as possible the cost of maintenance, the domes designers quickly passed from common structural steel to stainless steel, up to the present and widespread use of elements in aluminium alloys. Following this route, we arrive in the present day to the use of fiber-reinforced composite materials (textile materials).

2.1 The issue of the strength and the stability of lattice domes

In the modern lattice metal domes, with no hierarchy between elements of the same type and operating almost exclusively in extensional regime, the issue of the resistance of the different structural components (joints, bars and panels) is of secondary importance. At least during the last two centuries, the characteristics dimensions of the thin spherical domes have remained almost unchanged, as well as the intensity of the live loads (actions of wind and snow). On the other hand, the strength of the materials has increased considerably and it must be considered that, in the presence of an extensional stress state, the structural elements can carry out their resisting function at best. It therefore follows that, nowadays, other aspects are to be considered as fundamental for the search of an optimal structural solution.

The major issue that rules the choice of the particular lattice domes is without any doubt the stability of equilibrium. In fact, the increase in the strength of the new materials, and the small changes or even the lowering of the loads imposed on the structure, turned out in a progressive decrease in the size of the resistant elements. On the one hand this has resulted in a substantial economy in the use of materials, but on the other, being also decreased the stiffness of the elements, the structures have become more susceptible to buckling than in the past. The loss of stability can occur at the local level, if it involves single bars or panels, whenever to the latter are also assigned resisting functions, in addition to coating functions, or at the global level, if the phenomenon concerns one or more joints and the bars directly connected to them.

2.2 The spherical geodesic lattice domes

The search for technical solutions corresponding to a reduction of the costs of construction of the dome pushes very often to a standardization of the dimensions of the structural components. In this sense, Füller scored an extremely important result with the "*reticular geodesic dome*". The idea of Füller is to find a simple law that allows drawing a triangular mesh on a sphere (spherical grid). The method chosen is the projection on the spherical surface of the thirty sides of an icosahedron, concentric to the sphere. On each face of the original icosahedron is then traced a net with sides parallel or perpendicular to those of the same face. In both cases, the plane lattice thus obtained is projected radially from the centre of the icosahedron on the circumscribed sphere. This method of division has been the subject of a famous patent filed by the same Füller in December 1951 (Füller, [6]).

The result of this second subdivision and the subsequent projection is that the dimensions of the sides, as well as those of the faces, in which the sphere is now divided are no longer equal to each other, although such differences are still technically acceptable. The elements (bars/panels) placed near the vertices of the icosahedron will have minimal length/area, while those located near the centres of gravity of the faces might be larger; these regions are therefore equipped with lower stiffness.

The mesh of each spherical geodesic dome is chosen in such a way to reduce the variance of the length of the bars, as far as possible, by exploiting various symmetries. The most important are listed below.

a) "*binary symmetry*" or rotational symmetry with respect to each axis passing through the midpoint of two diametrically opposite sides of the icosahedron. The lattice overlaps with itself when it is rotated by 180° around each of those axis. This symmetry greatly simplifies the design of the spherical grid since there are 15 binary axes of symmetry.
b) "*ternary symmetry*" or rotational symmetry around each axis passing through the centre of gravity of two faces of the icosahedron that are perpendicular to it and diametrically opposed. The lattice overlaps with itself when it is rotated by 120° around each of those axis. Such symmetry is the more evident, since it allows to simply checking which elements of each face correspond to each other and, therefore, have same dimensions. Since there are 10 symmetry axes ternary, standardized

production of the elements is strongly favoured.

c) "*quinary symmetry*" or rotational symmetry around each axis passing through two diametrically opposite vertices of the icosahedron. The lattice overlaps with itself when it is rotated by 72° around each of those axis.

The presence of these symmetries not only decreases the number of different components (joints, bars, panels), but, above all, facilitates the automatic generation of the entire lattice, once provided the value of the radius of the sphere and of the parameters of tessellation. By this way, several spherical lattices can be compared with each other allowing to choose the one that suites best the design requirements of the dome.

3 A LATTICE OF INFLATABLE BEAMS

The technical solution for domes, which we intend to illustrate in its main features in this paper, is characterized by the use of inflatable elements as beams and membranes for the panels. This choice is motivated primarily by the opportunity to obtain by this way domes considerably lighter than those made with metallic beams, and that can be quickly built and removed without having to resort to cumbersome centrings or lifting devices. These characteristics represent as many advantages in the case of temporary constructions or whenever the dome has to be erected very quickly, as it occurs, for example, following a calamitous event. Other properties of some interest, which we will discuss in more detail in the following, concern the ultimate behaviour near collapse that is reasonably expected for this type of structures. The inflatable beams, as well as panels, are, in fact, in a state of pre-traction, induced by the internal pressure in the beams, which exerts a beneficial action on their load-bearing capacity and on the stability of the equilibrium configuration. Moreover, even once the limit load that corresponds to the onset of a collapse mechanism for the dome is reached, the dome will return in its initial configuration, occupied before the application of the load, if the load is removed (*reversible shakedown*).

4 A FIRST APPLICATIVE EXAMPLE: AN OPEN GEODESIC DOME

In order to illustrate by an example the proposed technical solution, let us consider the open geodesic dome shown in Figure 1.

The structure consists essentially of a thin hemispherical dome interacting with a triangular lattice of beams arranged in a single layer. The radius R, both of the membrane and the mean surface of the lattice of beams, is equal to 10.0 m.

The chosen geodesic dome is a simple layer one, characterized by a tessellation of type "*alternating parallel*" $\{3,5+\}_{10.0}$ having a triangulation number $T = 100$. If the lattice would cover the entire sphere, the number of nodes V would be equal to $V = 10T + 2 = 1002$; moreover, $E = 30T = 3000$ and $F = 20T = 2000$ would represent, respectively, the number of edges and the faces of the polyhedron (we recall that $F + V = S + 2$, according to Euler's formula for the uniform polyhedra). For reasons of space, here we omit the details that would justify the above-mentioned relations; for more details about the classification of the tessellation of a geodesic dome and about the geometric relationships existing between the triangulation number and the number of vertices of the polyhedron, we refer to (Ligarò, [7]). In the specific case, the dome being a semi-sphere, the vertices (nodes) reduce to $V = 526$, the edges (i.e., the bars) to E = 1525, and the panels (faces) to F = 1000.

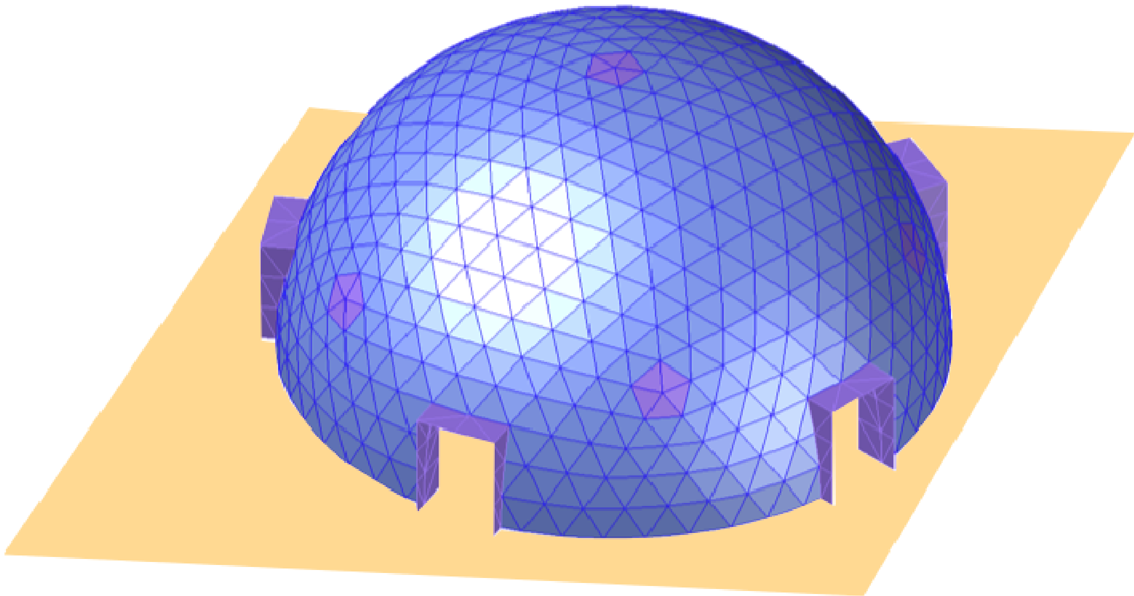


Figure 1: an open geodesic dome.

Both the bars and the panels are made of the same composite material. Each bar is a thin walled cylinder (diameter d = 12.0 cm, thickness t = 0.1 cm, average length h = 125 cm). The panels are plane triangular nearly equilateral membranes with thickness t = 0.1 cm. The composite material is formed by glass fibers (S-Glass) and a matrix thermoplastic resin (PVC), 70% in volume. In each thin structural element (i.e., wall tube or panel) a plane stress condition is assumed, the constitutive relation for the material is shown below:

$$\mathbf{e} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_1 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \mathbf{Ds}, \tag{1}$$

where $E_1 = E_2$ = 6110 kN/cm^2, G_{12} = 333,5 kN/cm^2, ν_{12} = 0,274. The composite material is assumed to have a termal expansion coefficient $\alpha_l = 1{,}38 \times 10^{-5}$ C^{-1} and a specific weight $\gamma = 1{,}961 \times 10^{-5}$ kN/cm^3.

In the following, the distribution of stresses in the beams and the panels of the geodesic dome are showed for different load conditions. For what concerns the initial optimization phase of the structure we refer to (Ligarò, [7]); the static analysis is carried out by using a common commercial finite element software, assuming a linear elastic behaviour of the structure. In particular, we consider the four elementary load conditions listed below:

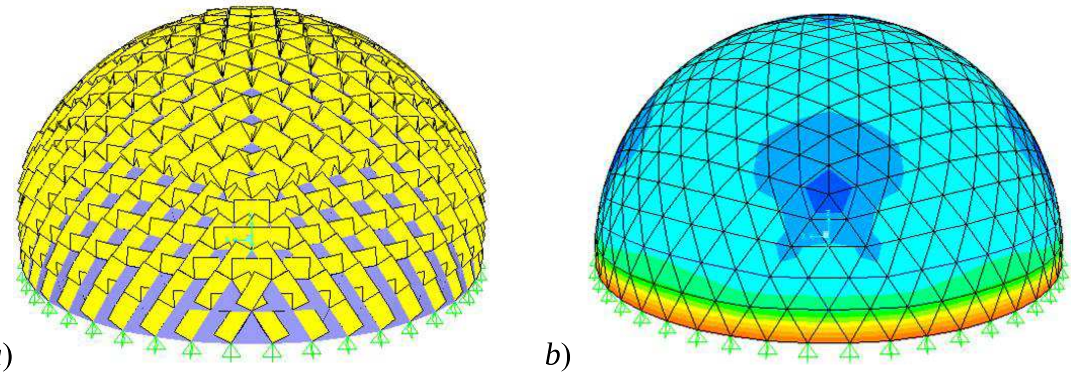
- inflation of beams;
- structure's self weight;
- snow load;

- wind load.

The final internal pressure of all the inflated beams is set equal to 2 bar. The static actions due to snow and wind were evaluated according to the technical regulations currently in force in Italy [8]. In this regard, it is assumed that the building is in a location at the sea level in the province of Pisa, in Tuscany.

4.1 The inflation phase

In the solution adopted, the dome is built by simply inflating the beams that make up the lattice up to reach an internal pressure of 0.02 kN/cm^2. The distribution of the axial force in the beams and that of the minimum principal stress in the triangular panels is shown in Figure 2.



a) *b*)

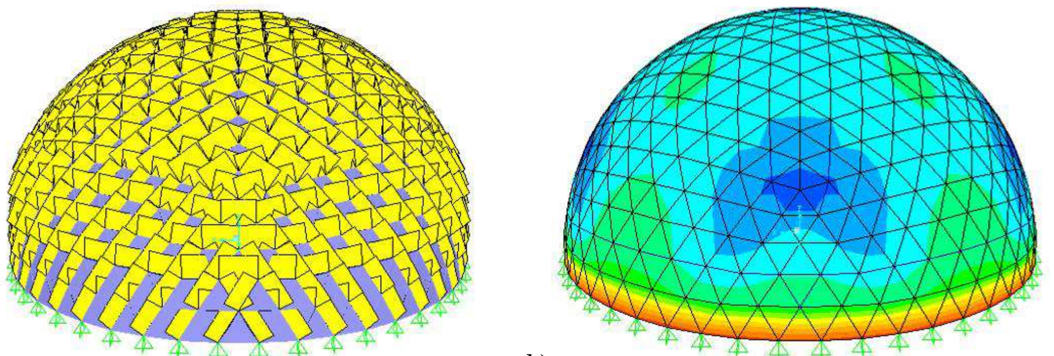
Figure 2: *a*) axial force in the inflated beams (min = 0,54 kN; max = 0,87 kN); *b*) minimum principal stress in the shell elements (min = 0.03 kN/cm^2; max = 0.28 kN/cm^2).

The figure clearly shows that the regularity of the tessellation allows for obtaining a state of pre-tension in the beams, which is characterized by a variability contained within more than acceptable limits (the minimum axial force is equal to 60% of the maximum). The pre-traction is almost uniform for triangular panels also, except for the layer that is directly in contact with the fixed constraints placed at the base of the hemisphere.

With reference to this first phase, it has also to be noted that the average value of the tensile stresses in the panels is equal to about 0.46 kN/m, and that the work required for the inflation of the structure is equal to about 8480 kNm. By way of example, we observe that the work of inflation would be considerably less (of the order of 1000 kNm) if a uniform thin shell membrane would be put under pressure from the inside of the dome, while keeping the constant the average stress in the panels. In other words, for a fixed stress level in the material, the proposed solution allows storing a much larger amount of elastic energy in the elements of the dome, compared to the so-called pneumatic constructions.

4.2 The dome self-weight

The stresses in the structure that are produced by the combined action of the dome self-weight and the pre-traction state induced by inflating of the beams are shown in Figure 3.



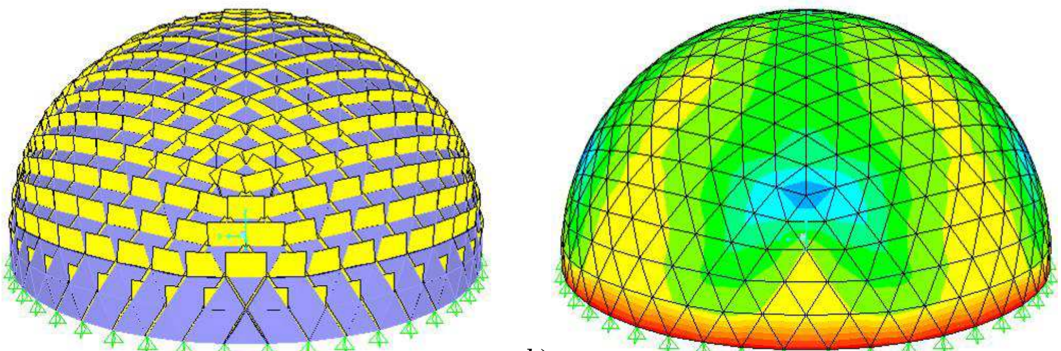
a) *b*)

Figure 3: *a*) axial force in the inflated beams (min = 0,48 kN; max = 0,87 kN); *b*) minimum principal stress in the shell elements (min = 0.02 kN/cm^2; max = 0.27 kN/cm^2).

Given the extreme lightness of the dome, the effect of the weight of the structure produces, as was expected, only slightly appreciable changes in the distribution of the stresses with respect to the previous case. It has also to be noted that the small value of the thickness of the panels (t = 0.1 cm) inevitably leads to consider the typical problem of stress concentration (localized reinforcement), as commonly happens near the edges of any tensile structure.

4.3 The snow load

If the stresses assessed in the previous section are added to those due to snow load, we obtain the results shown in Figure 4. As regards the magnitude of the load, this was evaluated according to the NTC2008. In particular, it was assumed that the building is placed at the sea level, in the province of Pisa (therefore, it falls within the zone III), and that the exposure coefficient is equal to unity. Under these conditions, the design value of the snow load is equal to 0.48 kN/m^2.



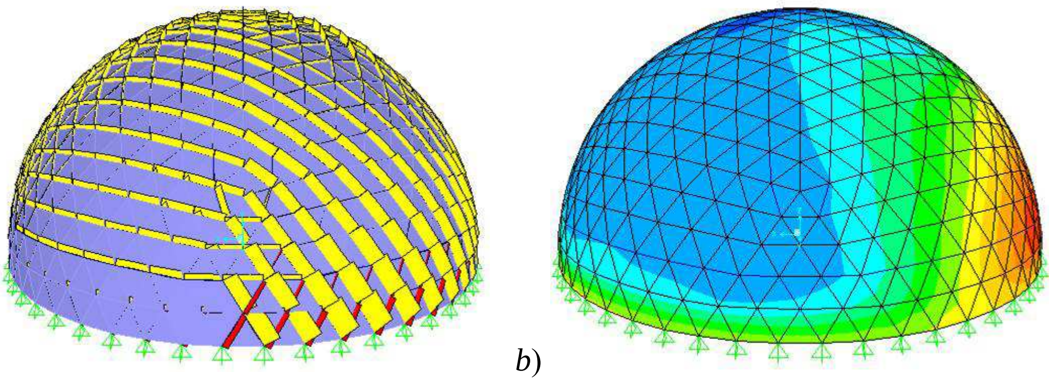
a) *b*)

Figure 4: *a*) axial force in the inflated beams (min = -0,17 kN; max = 1,65 kN); *b*) minimum principal stress in the shell elements (min = -0.07 kN/cm^2; max = 0.11 kN/cm^2).

It is noted that the snow load, when it reaches the design value, causes the onset of compressive stresses in the inflated beams. Since in the model we have adopted the beams are not able, by hypothesis, to support any compressive stress, it follows that already at this load level the structure will exhibit a mechanical response of non-linear type, although scarcely perceptible. In this regard, we observe, in fact, that the percentage of the beams subject to an appreciable compression (say higher, in absolute value, to 0.1 kN) is equal to 4%. The same percentage would drop to one per cent if the magnitude of the snow load were reduced by 10 %. Even the triangular panels are subject to tensile stresses almost everywhere, except the narrow band directly in contact with the constraints placed at the base of the dome.

4.4 The wind load

If the stresses evaluated in section 4.2 (weight + inflation) are added to those produced by the wind, we obtain the results shown in Figure 5. As regards the magnitude of the load, this was evaluated according to the NTC2008. In particular, it is assumed that the construction is in Tuscany, and therefore in the area 3, that the category of exposure is the third and that the roughness class of the soil is B (the one typical, for example, of the urban areas). Under these conditions, the design value of the wind load is equal to 0.46 kN/m^2. To determine the pressure on the generic element of the dome, the design value of the uniform distributed load is multiplied for the coefficients of exposure and shape, C_e and C_p. The value attributed to these coefficients depends on the particular inclination of the cover: the corresponding values of C_e and C_p were calculated for each panel before performing the static analysis.



a) *b*)

Figure 5: *a*) axial force in the inflated beams (min = -0,17 kN; max = 1,65 kN); *b*) minimum principal stress in the shell elements (min = -0.07 kN/cm^2; max = 0.11 kN/cm^2).

As it was expected, the wind is the more engaging load for the dome structure. The first observation, which clearly emerges from Figure 5, is the presence of some compressed beams and panels. This result is clearly inconsistent with the particular type of structure we have chosen to use. The inflatable beams can withstand only very small compressive stresses that, in any case, are negligible compared to tensile stresses. As a consequence, an incremental stress analysis should be performed: once a beam becomes unstressed it should not be considered in the subsequent load increments. By this way, it is reasonable to believe that the distribution of stresses would show some differences, although not so large, with respect to that shown in Figure 5. In particular, the lattice of beams, as well as the set of panels, would present an inactive part, where the elements are unstressed, and an active (tense) part. Moreover, the tractions will be greater than those calculated in the linear scheme that we have adopted.

A second observation, contrarily to the first one, makes reasonable assuming less noticeable differences between the linear and nonlinear solution. In the dome region exposed over the wind, the pressure may be represented as a system of inward pressing forces on the nodes of the lattice. The beams converging at any one of these nodes will be subjected to a state of compression. However, as soon as the first compressive stresses appear, the stiffness of the inflatable beams reduces considerably until it becomes evanescent, and the node becomes free to move in the direction of the force acting on it. The displacement of the node continues until the local curvatures of the dome reverse their sign and the beams become stretched (anticlastic curvature), thus regaining their stiffness (Pomeroy, [9]). In other words, by taking into account the above-described phenomena, the compression obtained by the linear analysis may be considered, in some sense, as "*apparent*", since they could be substituted by tensile stresses, at least in some cases, if a nonlinear analysis would be executed.

Finally, we observe that the dome might not be perfectly airtight for strong winds. In these cases, the intensity of the resulting wind pressure on the dome reduces considerably.

The severity of the state of stress produced by the wind suggests the opportunity, however, to make some changes in the overall organization of the structure, by adopting suitable measures able to reduce the effects of such actions. In this regard, a first simple solution might be to differentiate the intensity of the inflation pressure according to the position of the beams.

5 CONCLUSIONS

- In this work we shown a first example of spherical geodesic dome made by a network of medium pressurized inflatable beams, interacting with an elastic membrane shell.
- The design problem of the dimensioning of the elements that make up the dome has requested, to be solved, the analysis of different topics. A first basic item is represented by the choice of the particular network of beams. Here we have chosen a suitable tessellation (alternating parallel) defined by the number of triangulation. The geometry of the lattice was found by trying to make minimum, as much as possible, the variance of the length of the bars.
- The main characteristic feature of this study is the proposal to use light inflatable beams made of composite material instead of the usual metal elements. This choice allows obtaining constructive solutions of lower weight, which may be built-up and removed quickly, without having to resort to cumbersome centrings or lifting devices. The inflatable beams, as well as the panels, are in a state of pre-traction, induced by the internal pressure in the beams, which exerts a beneficial action on the load bearing capacity and on the onset of buckling phenomena.
- Even if the limit load for the dome is reached, and a collapse mechanism is activated, the dome will return to its initial configuration once the load is removed. Finally, the lattice of inflatable beams is considerably less vulnerable to incidental damage, compared to the solutions of the pneumatic type. The final example has shown the possibilities offered by this type of technical solution.

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